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Some Qualitative Considerations on the Numerical Determination of Minimum Mass Structures with Specified Natural Frequencies

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Some Qualitative Considerations on the Numerical Determination of Minimum Mass Structures with Specified Natural Frequencies[1,2]

by

A. Mangiavacchi[3] and A. Miele[4]

Abstract. The problem of the axial vibration of a cantilever beam is investigated analytically. The range of values of the frequency parameter having technical interest is determined.

Key Words. Structural optimization, cantilever beams, axial vibrations, fundamental frequency constraint.

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[2]The authors are indebted to Dr. V.B. Venkayya, Wright-Patterson AFB, Ohio, for suggesting the topic.

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Notation

E	Modulus of elasticity, lb ft^{-2}
L	Length of the beam, ft
m	Normalized mass per unit length, $m = ML/M_o$
M	Mass per unit length, lb ft^{-2} sec^2
M_o	Reference mass, lb ft^{-1} sec^2 (Sections 2-3)
M_o	Tip mass, lb ft^{-1} sec^2 (Sections 4-5)
M_*	Total mass of the beam, lb ft^{-1} sec^2
x	Normalized axial coordinate, $x = X/L$
X	Axial coordinate, ft
u	Normalized axial displacement, $u = Y(X)/Y(L)$
Y	Axial displacement, ft
β	Frequency parameter, $\beta = \omega L\sqrt{(\rho/E)}$
ρ	Density, lb ft^{-4} sec^2
ω	Natural frequency, sec^{-1}

Superscript

$'$	Derivative with respect to the normalized axial coordinate x (for example, $u' = du/dx$)

1. Introduction

In this memorandum, we consider the problem of the axial vibration of a cantilever beam. With reference to a constant-section beam, we determine the range of values of the frequency parameter β having technical interest. This range of values of the frequency parameter is important in the solution of a subsequent problem: the determination of the mass distribution that minimizes the total mass of a beam for a given fundamental frequency constraint.

2. Nonoptimal Beam without a Concentrated Mass

Let m denote the normalized mass per unit length, u the normalized axial displacement, and β the frequency parameter. Let x denote the axial coordinate, normalized so that x=0 at the base of the beam and x=1 at the tip of the beam. Let the prime denote total derivative with respect to the axial coordinate x. With this understanding, the fundamental equation to be solved is the following:

$$(mu')' + \beta^2 mu = 0 . \tag{1}$$

In this equation, the frequency parameter β is related to the natural frequency ω, the length L, the density ρ, and the modulus of elasticity E by the relation

$$\beta = \omega L\sqrt{(\rho/E)} . \tag{2}$$

In the absence of a concentrated mass attached at the tip of the beam, the boundary conditions for Eq. (1) are as follows:[5]

$$u(0) = 0, \quad m(1)u'(1) = 0 . \tag{3}$$

If the mass distribution

$$m = m(x) \tag{4}$$

is prescribed a priori, then (1) is a second-order differential

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[5]Equations (3) must be completed by the normalization condition $u(1) = 1$.

equation, to be solved in conjuction with the boundary conditions (3).

Constant Section. Next, we consider the particular case of a constant-section structure, that is, a structure with a constant mass per unit length:

$$m = \text{const} . \tag{5}$$

For this particular case, the differential equation (1) and the boundary conditions (3) simplify as follows:

$$u'' + \beta^2 u = 0 , \tag{6}$$

$$u(0) = 0, \quad u'(1) = 0 . \tag{7}$$

The solution of (6) consistent with the initial condition (7-1) is the following:[6]

$$u = A \sin(\beta x) , \tag{8}$$

with the implication that

$$u' = A\beta \cos(\beta x) . \tag{9}$$

From (9) and the final condition (7-2), we conclude that

$$\cos \beta = 0 , \tag{10}$$

so that

$$\beta = (2n+1)\pi/2, \quad n = 0,1,2,... . \tag{11}$$

[6]The constant A has the value $A = 1/\sin\beta$.

Therefore, for this problem, the smallest nontrivial value of the frequency parameter is

$$\beta = \pi/2 \ . \tag{12}$$

3. Optimal Beam without a Concentrated Mass

Now, suppose that a constant-section structure has been studied in accordance with Section 2. Suppose that the frequency parameter β which allows satisfaction of the boundary conditions (7) has been determined, namely, $\beta=\pi/2$. The total mass of the structure studied in Section 2 is given by[7]

$$M_*/M_o = \int_0^1 m dx , \qquad m = \text{const} . \tag{13}$$

Therefore, it is natural to pose the following question: for the same value of the frequency parameter $\beta=\pi/2$, is there a better beam, that is, one having a smaller total mass? In particular, is there a beam which yields the smallest total mass for the given value of β? This question leads to the following variational problem: Minimize the total mass

$$M_*/M_o = \int_0^1 m dx , \qquad m = m(x) , \tag{14}$$

with the understanding that the following constraints must be satisfied:[8]

$$(mu')' + \beta^2 mu = 0 , \tag{15}$$

$$u(0) = 0 , \quad m(1)u'(1) = 0 , \tag{16}$$

and with the further understanding that $\beta=\pi/2$. Owing to

the fact that the problem (15)-(16) is homogenous, the obvious solution under the physical constraint

$$m(x) \geq 0 \tag{17}$$

is

$$m(x) = 0 , \tag{18}$$

with the implication that

$$M_*/M_o = 0 . \tag{19}$$

In order to avoid the occurrence of the above trivial solution, Ineq. (17) could be changed as follows:

$$m(x) \geq m_o . \tag{20}$$

Then, the solution would become

$$m = m_o . \tag{21}$$

To arrive at solutions other than constant mass solutions, it is necessary to postulate some different physical situation (e.g.,a concentrated mass attached at the end of the beam). In turn, this results in a change in the boundary condition (16-2), and this change makes it unnecessary to employ inequality constraints of the form (17) or (20).

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[7] The symbol M_o denotes a reference mass.

[8] Equations (16) must be completed by the normalization condition $u(1) = 1$.

4. Nonoptimal Beam with a Concentrated Mass

In this section, we assume that a concentrated mass M_o is attached at the tip of the beam. Using the same terminology as in Section 2, we see that the governing differential equation (1) still holds:

$$(mu')' + \beta^2 mu = 0 . \tag{22}$$

On the other hand, the boundary conditions (3) are modified as follows:[9]

$$u(0) = 0, \quad m(1)u'(1) = \beta^2 . \tag{23}$$

Constant Section. Again, we consider the particular case of a constant-section structure. Under condition (5) and after observing that

$$M_*/M_o = m , \tag{24}$$

then problem (22)-(23) becomes

$$u'' + \beta^2 u = 0 , \tag{25}$$

$$u(0) = 0, \quad u'(1) = (M_o/M_*)\beta^2 . \tag{26}$$

The solution of (25) consistent with the initial condition (26-1) is the following:

[9] Equations (23) must be completed by the normalization condition $u(1) = 1$.

$$u = A \sin(\beta x), \tag{27}$$

with the implication that

$$u' = A\beta \cos(\beta x) . \tag{28}$$

From (28) and the final condition (26-2), we conclude that

$$A \cos \beta = (M_o/M_*)\beta . \tag{29}$$

Owing to the fact that

$$u(1) = A \sin \beta , \tag{30}$$

elimination of A from (29)-(30) leads to the following transcendental equation:

$$\beta \tan \beta = (M_*/M_o)u(1), \tag{31}$$

which, for u(1)=1, reduces to

$$\beta \tan \beta = M_*/M_o . \tag{32}$$

This equation supplies the frequency parameter β in terms of the mass ratio (ratio of beam mass M_* to tip mass M_o).

In order to understand the significance of (32), let us consider two limiting cases: (i) negligible mass ratio and (ii) infinite mass ratio. If $M_*/M_o = 0$, then the solution of (32) is

$$\beta = n\pi, \quad n = 0,1,2,... . \tag{33}$$

On the other hand, if $M_*/M_o = \infty$, then the solution of (32) is

$$\beta = (2n+1)\pi/2 , \qquad n = 0,1,2,\ldots , \tag{34}$$

which is identical with (11). Since the first natural frequency corresponds to $n = 0$, we conclude that, for mass ratios in the range

$$0 \leq M_*/M_o \leq \infty , \tag{35}$$

the smallest frequency parameter β consistent with the trascendental equation (32) lies in the range

$$0 \leq \beta \leq \pi/2 . \tag{36}$$

5. Optimal Beam with a Concentrated Mass

As in Section 3, we can formulate the problem of finding the optimal mass distribution. The problem is as follows: Minimize the total mass

$$M_*/M_o = \int_0^1 m dx\,, \qquad m = m(x)\,, \tag{37}$$

with the understanding that the following constraints must be satisfied:[10]

$$(mu')' + \beta^2 mu = 0\,, \tag{38}$$

$$u(0) = 0\,, \quad m(1)u'(1) = \beta^2\,, \tag{39}$$

and with the further understanding that the frequency parameter β has some fixed value in the range

$$0 \leq \beta \leq \pi/2\,. \tag{40}$$

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[10] Equations (39) must be completed by the normalization condition $u(1) = 1$.

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